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AN IMPROVED METHODOLOGY FOR RELIABILITY GROWTH PROJECTIONS

LARRY H. CROW

JUNE 1982

U. S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
ABERDEEN PROVING GROUND, MARYLAND

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At the end of a development test phase delayed fixes are often incorporated into the system and it is usually desirable to make a projection of the impact of these fixes on the system reliability at the beginning of the next phase of development. The most common procedure in practice for making reliability projections utilizes engineering assessments of the effectiveness of the delayed fixes for each observed problem failure mode. These effectiveness factors are then used with data generated during the test phase to obtain a projected

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estimate for the updated configuration by adjusting the number of failures observed during the test phase. We call this method the "adjustment procedure" (AP).

In this report, we study the accuracy associated with the AP for reliability projections. Results of this study show that there are two sources of error associated with the AP. The consequence of these errors is that the AP is overly optimistic, generating projected values which, on average, over-estimate the system reliability. In particular, it was found that even when the effectiveness factors are known exactly, the AP will still overestimate the system reliability.

A procedure is developed in this report for estimating the error term of the AP. Utilizing this procedure a practical projection methodology is constructed which gives improved accuracy for making reliability projections. Numerical examples illustrating the application of this methodology are given.

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AN IMPROVED METHODOLOGY FOR RELIABILITY GROWTH PROJECTIONS

1. INTRODUCTION

The need for highly reliable military systems is obvious and this need is generally reflected in high reliability requirements to be attained during development. The early prototypes for complex, military systems will invariably have significant reliability and performance deficiencies and, consequently, these systems are subjected to a development testing program to find problems and take corrective action. The improvement in reliability and performance which occurs will depend on the number and the effectiveness of the fixes that are incorporated into the system.

Experience has shown that programs which rely simply on a demonstration test by itself to determine compliance with the reliability requirements generally do not achieve the reliability objectives with the allocated resources. It has become increasingly clear that management and engineering attention to reliability throughout the program is necessary if the high requirements for complex systems are to be met. Reliability growth management is defined in US Department of Defense Handbook 189 as "the systematic planning for reliability achievement as a function of time and other resources, and controlling the on-going rate of achievement by reallocation of resources based on comparisons between planned and assessed reliability values."

Major management decisions regarding the reliability effort are made based on comparisons of the assessed and target values. If the assessments are in agreement with the target values, the reliability program typically will remain unchanged. However, if the assessed values are well below the target values, then major changes in the program may be necessary. If the assessed values do not accurately reflect the system reliability status, then clearly, incorrect management decisions can be made; the system may be accepted and fielded with lower reliability than desired or unnecessary and costly changes and delays in the program may occur.

For most development programs, reliability assessment methodologies must account for a dynamic environment due to modifications being incorporated into the system. For example, in the presence of reliability growth, the data from the earlier part of the test phase would not be representative of the current configuration. On the other hand, the most recent test data, which would best represent the current configuration, may be limited, and not in itself be sufficient for a valid reliability estimate. Because of this situation, various growth models or techniques are often employed for assessing system reliability during a test phase.

According to the format in US Department of Defense Handbook 189, two reliability estimates would typically be made at the end of a test phase. The demonstrated reliability value is an estimate of the system reliability for its configuration at the end of the test phase. The demonstrated value is based on data generated during the test phase. Also, at the end of the test phase delayed fixes are often incorporated into the system and it is usually desirable to make a projection of the impact of these fixes on the system reliability at the beginning of the next phase of development.

Most of the reliability growth literature has been concerned with procedures and models most appropri ... a demonstrated reliability value and very little attention has been p... to techniques for reliability projections based on delayed fixes. The most common procedure, in practice, for making reliability projections when fixes are delayed until the end of the test phase, utilizes engineering assessments of the effectiveness of the delayed fixes for each observed problem failure mode. The effectiveness factors are then used with data generated during the test phase to obtain a projected estimate for the updated configuration by adjusting the number of failures observed during the test phase. We call this method the "adjustment procedure." See US Department of Defense Handbook 189 for a discussion of reliability growth projection procedures.

In this report, we study the accuracy associated with the adjustment procedure and propose an improved projection model. Specifically, in Section 2 we define a rigorous structure and framework, consistent with real life, for investigating the projection procedures. Based on this structure, we show that there are two sources of statistical bias associated with the adjustment procedure which generally cause one to overestimate the system reliability. In particular, it is shown that even when the effectiveness factors are known exactly, the adjustment procedure is still a biased estimate. In Section 3 we develop an improved projection model which removes one important source of error inherent in the adjustment procedure.

2. COMMENTS ON THE ADJUSTMENT PROCEDURE

Before we describe the adjustment procedure, we will first give some background and establish some needed notation.

Suppose a system is subjected to development testing for a period of time T. The system can be considered as consisting of two types of failure modes. Type A modes are all failure modes such that when seen during test, no corrective action will be taken. This accounts for all modes for which it is not cost-effective to attempt to increase the reliability by a design change. Type B modes are all modes such that if seen, a design change, or fix, will be attempted.

It is assumed that all Type B modes are in series and fail independently according to the exponential distribution. We also assume that the occurrence of Type A modes follow the exponential distribution with failure rate λ_A. In this paper, we will assume that fixes for Type B modes found during test will be incorporated as delayed fixes at the end of the test phase.This implies that the system reliability is constant throughout the test phase and will, then, jump to a higher value after the delayed fixes have been implemented.

Let K denote the number of Type B modes in the system and let λ_i be the failure rate for the i-th Type B mode, i = 1,...,K. Then, at time 0, the system failure rate r(0) is

$$r(0) = \lambda_A + \lambda_B, \qquad (1)$$

where $\lambda_B = \sum_{i=1}^{K} \lambda_i$. (In practice, of course, K will generally not be known before or after the testing.)

During the test time (0, T) a random number $M \le K$ of distinct Type B modes will be observed. We further denote by d_i the effectiveness factor (EF) for the i-th Type B mode, i=1,...,K. The factor d_i is the percent decrease in λ_i after a corrective action has been made for the i-th Type B mode.

Let $i_1, i_2, \ldots, i_M$ be random variables which denote the indices of the Type B modes observed during the test time (0,T). As a result of the M corrective actions taken on the M Type B modes observed, the system failure rate at time T is reduced from r(0) to

$$r(T) = \lambda_A + \sum_{j=1}^{M} (1 - d_{i_j})\lambda_{i_j} + (\lambda_B - \sum_{j=1}^{M} \lambda_{i_j}) \tag{2}$$

$$= \lambda_A + \lambda_B - \sum_{j=1}^{M} d_{i_j}\lambda_{i_j}. \tag{3}$$

The term

$$\sum_{j=1}^{M} (1 - d_{i_j})\lambda_{i_j} \tag{4}$$

is the failure rate for the M modes after the corrective actions. The term

$$(\lambda_B - \sum_{j=1}^{M} \lambda_{i_j}) \tag{5}$$

is the remaining failure rate for all unseen Type B modes.

The adjustment procedure is a method which has been used as an estimate of r(T). We will show that this procedure is not valid, and that it will usually yield erroneous, totally misleading results. In particular, we show that the adjustment procedure will generally overestimate the true current system reliability.

Let N_A, N_B be the total number of Type A and Type B failures observed and let $N = N_A + N_B$. For the $M \le N_B$ distinct Type B modes observed during test we let N_{i_j} denote the number of observed failures for B mode i_j. The adjustment procedure consists of reducing the N_{i_j} for the observed Type B modes to reflect a decrease in failure rate resulting from the correction action. Let $d^*_{i_j}$ denote the assumed EF. Then, the adjustment procedure modifies N by

$$N^* = N_A + \sum_{j=1}^{M} (1 - d^*_{i_j})N_{i_j} \tag{6}$$

and estimates the system failure rate at time T by

$$r^*(T) = N^*/T \tag{7}$$

$$= N_A/T + \sum_{j=1}^{M} (1-d^*_{i_j})\, N_{i_j}/T. \tag{8}$$

Observe that if $d^*_{i_j} = 1$ (i.e., the fixes are all assumed to be 100 percent effective), then

$$r^*(T) = N_A/T \tag{9}$$

which simply estimates the failure rate for the Type A modes. This will certainly underestimate the system failure rate at time T unless all problem failure modes (Type B) in the system have been found and completely removed. This rarely, if ever, happens for complex systems during development.

To quantify the bias for the adjustment procedure we next consider the expected value of $r(T) - r^*(T)$. Observe that $r(T)$, the true system failure rate at time T, is a random variable depending on the Type B modes seen during the test and their EFs.

Let $I_i(t)$ denote the indicator function defined by

$$I_i(t) = \begin{cases} 1 & \text{if } i\text{-th Type B mode occurs during } (0,t) \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

Then, we may write $r(T)$ as

$$r(T) = \lambda_A + \sum_{i=1}^{K} [1 - d_i I_i(T)]\, \lambda_i\,. \tag{11}$$

Also, note that $r^*(T)$ may be written as

$$r^*(T) = N_A/T + \sum_{i=1}^{K} (1 - d^*_i)\, N_i/T. \tag{12}$$

Therefore,

$$E(r^*(T)) = E(N_A/T + \sum_{i=1}^{K} (1-d^*_i)\, N_i/T) \tag{13}$$

$$= \lambda_A + \sum_{i=1}^{K} (1 - d^*_i)\, \lambda_i \tag{14}$$

and

$$E(r(T)) = E(\lambda_A + \sum_{i=1}^{K} [1 - d_i I_i(T)] \lambda_i) \tag{15}$$

$$= \lambda_A + \sum_{i=1}^{K} [1 - d_i(1 - e^{-\lambda_i T})] \lambda_i. \tag{16}$$

This yields

$$E(r(T)-r^*(T)) =$$

$$\sum_{i=1}^{K} (d_i^* - d_i)\lambda_i + \sum_{i=1}^{K} d_i \lambda_i e^{-\lambda_i T}. \tag{17}$$

Equation (17) gives the average difference in the adjustment procedure estimate and the actual system failure rate. The first term in (17) is, of course, the contribution to the bias resulting from the difference between the estimated EFs and the true EFs. The second term is the contribution to the bias resulting from the randomness associated with the occurrence of Type B problem modes during the test period (0,T).

Now, it is very important to note that $E(r^*(T))$ given by Equation (14) does not depend on T. That is, the expected value of the adjustment procedure estimate is a constant for all T. However, the expected value of the actual system failure rate given by Equation (16) decreases as the test time T increases. As a matter of fact, $E(r(T))$ approaches $E(r^*(T))$ as $T \to \infty$ for $d_i^* = d_i$. The adjustment procedure estimate $r^*(T)$ when $d_i^* = d_i$ is actually an estimate of the limiting system failure rate when all Type B modes have been observed and a fix incorporated. This implies that for $d_i^* = d_i$, $r^*(T)$ is an estimate of the lower bound $E(r^*(T))$ on system failure rate and is not a valid estimation procedure for $r(T)$, the current system failure rate.

Assume that $d_i^* = d_i$ and note that

$$E(r^*(T)) = \lambda_A + \lambda_B - \sum_{i=1}^{K} d_i \lambda_i. \tag{18}$$

The following remarks are to further clarify the significance of the bias associated with the adjustment procedure. The initial failure rate for the i-th Type B mode is λ_i. If the i-th mode is observed, the value that the failure rate will be reduced to is $(1-d_i)\lambda_i$ after the fix; that is, λ_i is reduced by the amount $d_i\lambda_i$. The probability that the i-th mode will be seen by time T is $1-e^{-\lambda_i T}$ and, consequently, the average amount λ_i is actually reduced by time T is

$$d_i\lambda_i(1 - e^{-\lambda_i T}). \tag{19}$$

From (18) we see that at time T the adjustment procedure decreases λ_i by the average amount

$$d_i\lambda_i. \tag{20}$$

To correct for this bias we, therefore, need to add the amount

$$d_i\lambda_i e^{-\lambda_i T} \tag{21}$$

to the adjustment procedure estimate for the i-th mode. The total amount of correction needed for all K modes is the bias term B(T), where we define B(t), $o \leq t \leq T$, by

$$B(t) = \sum_{i=1}^{K} \lambda_i d_i e^{-\lambda_i t} . \tag{22}$$

In the next section, we will develop a methodology for estimating the bias term B(T) which is appropriate when K, the number of problem failures modes, is large. This estimate of the bias will be used to construct a projection procedure which is, for all practical purposes, unbiased when d_i^* equals d_i. In this regard, the US Army Materiel Systems Analysis Activity (USAMSAA) has conducted studies to determine from historical data the actual EFs for helicopters, tanks, missiles and electronic equipment (see Trapnell, 1982). These factors, which are based on historical experiences, may be used as guidelines for assigning EFs to similar systems under development.

3. A SYSTEM RELIABILITY PROJECTION MODEL

In this section, we present a procedure for estimating the bias term B(T) given in Equation (22). The estimated bias will then be used to develop a reliability projection methodology which is appropriate under assumptions which are generally reasonable in practice for complex system development.

We let M(t) denote the random number of distinct Type B modes observed during (0,t) and observe that M(t) may be written as

$$M(t) = \sum_{i=1}^{K} I_i(t). \tag{23}$$

Hence,

$$E(M(t)) = \sum_{i=1}^{K} (1 - e^{-\lambda_i t}). \tag{24}$$

We denote by W(t) the cumulative sum of the EFs for the Type B modes observed during (0,t). It follows, of course, that

$$W(t) = \sum_{i=1}^{K} d_i I_i(t) \tag{25}$$

and

$$E(W(t)) = \sum_{i=1}^{K} d_i(1-e^{-\lambda_i t}). \tag{26}$$

Taking the derivative of E(W(t)) yields

$$(d/dt)\ E(W(t)) = \sum_{i=1}^{K} d_i\lambda_i e^{-\lambda_i t}. \tag{27}$$

Thus, the key result that

$$B(t) = d/dt\ E(W(t)). \tag{28}$$

To develop an approach for estimating B(t) we use the relationship (28) and develop a model which expresses E(W(t)) as a power function of t, namely,

$$E(W(t)) = \alpha t^{\beta} \tag{29}$$

where $\alpha > 0$, $\beta > 0$. With this expression for E(W(t)), we have

$$B(t) = \alpha\beta t^{\beta-1}. \tag{30}$$

To formulate the model we begin by noting that

$$h(t) = d/dt\ E(M(t)) = \sum_{i=1}^{K} \lambda_i e^{-\lambda_i t} \tag{31}$$

is the average rate with which a new Type B mode will occur at time t. We will motivate the non-homogeneous Poisson process with rate h(t) as a stochastic model to approximate the occurrence of new Type B modes. This model implies that $h(t)\Delta t$ is interpreted as the unconditional probability of a new Type B mode occurring in the interval $(t,t+\Delta t)$, instead of an average probability.

Consider an interval $(t,t+\Delta t)$ and let $M(t,t+\Delta t) = M(t+\Delta t)-M(t)$ denote the number of distinct Type B modes observed in this interval. Note that $M(t,t+\Delta t)$ is the sum of K independent Bernoulli random variables. Therefore, for $E(M(t,t+\Delta t)) = E(M(t+\Delta t))-E(M(t))$ small, and K large, it follows (See Feller, 1957, Ch. XI) that the distribution of $M(t,t+\Delta t)$ is approximately Poisson distributed.

For Δt infinitesimally small, $E(M(t,t+\Delta t))$ is approximately equal to $h(t)\Delta t$. Thus, for Δt infinitesimally small, the Poisson probability of at least one Type B mode occurring in $(t,t+\Delta t)$ is approximately $h(t)\Delta t$, and the Poisson probability of more than one distinct Type B mode

occurring in this interval is near zero. In addition, for large K, it is reasonable to assume that the number of distinct Type B modes observed in nonoverlapping intervals are independent. Under these conditions, it follows that the occurrence of distinct Type B modes are in accordance with the nonhomogeneous Poisson process with mean value function (24) and intensity function or rate, $h(t)$.

Based on empirical studies at USAMSAA on complex weapon systems (see Reiher, et al., 1978, page 49), it has been found that the mean number of new Type B modes by time t is often approximated well by the power function

$$E(M(t)) = \lambda t^{\beta}. \tag{32}$$

where $\lambda > 0$, $\beta > 0$. With this relationship, we have

$$h(t) = \lambda \beta t^{\beta-1}. \tag{33}$$

Now, $W(t)$ is the random sum of the EFs for those Type B modes observed during $(0,t)$. Within a test phase, it would appear reasonable to assume that the observed EFs may be treated as independent random variables, distributed around a common mean, say, μ_d, and independent of $M(t)$. Under these assumptions, we have

$$E(W(t)) = \mu_d \, E(M(t)) \tag{34}$$

or

$$E(W(t)) = \mu_d \lambda t^{\beta} = \alpha t^{\beta} \tag{35}$$

as in (29), where $\alpha = \lambda \mu_d$. Consequently, it follows that the bias term $B(T)$ is expressed as

$$B(T) = \alpha \beta T^{\beta-1}. \tag{36}$$

The mean μ_d is estimated by

$$\hat{\mu}_d = \frac{1}{M(T)} \sum_{j=1}^{M(T)} d_{i_j} \tag{37}$$

where the d_{i_j}'s are the EFs for the $M(T)$ modes observed. To estimate the parameters λ and β for the rate of occurrence of new Type B modes, one may use the method of maximum likelihood (ML). These estimates are given in Crow (1974).

Let $X_1 < X_2 < \ldots < X_M < T$ denote the cumulative test times for the first occurrences of Type B modes, where we let $M(T) = M$. Then, the ML estimates of β and λ are

$$\hat{\beta} = \frac{M}{\sum_{i=1}^{M} \log(T/X_i)} \tag{38}$$

and

$$\hat{\lambda} = \frac{M}{T^{\hat{\beta}}} \quad (39)$$

The intensity function h(t) is estimated by

$$h(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1} \quad (40)$$

for $t > 0$. In particular the ML estimate for the rate of occurrence for distinct Type B modes at time T is

$$\hat{h}(T) = \hat{\lambda}\hat{\beta}T^{\hat{\beta}-1} = \frac{M\hat{\beta}}{T} \quad . \quad (41)$$

Further, the ML estimate of the bias term B(T) is given by

$$B(T) = \hat{\mu}_d \frac{M\hat{\beta}}{T} \quad . \quad (42)$$

Note that the ML estimate of α is

$$\hat{\alpha} = \hat{\lambda}\hat{\mu}_d. \quad (43)$$

Example

In this example, we illustrate the ML procedure for estimating the occurrence rate h(t) for new Type B modes. Suppose a system were tested for T = 400 hours and 15 distinct Type B modes were first observed at the following cumulative test times: 0.2, 11.2, 37.2, 39.0, 48.4, 53.4, 90.2, 91.6, 151.4, 159.4, 197.2, 240.2, 323.6, 361.2, 381.6. These data were generated by computer simulation of the non-homogeneous Poisson process with $h(t) = \lambda\beta t^{\beta-1}$ and $\lambda = 0.42$, $\beta = 0.5$. From Equations (39) and (38), the ML estimates of λ and β are determined to be $\hat{\lambda} = 0.501$, $\hat{\beta} = 0.567$. Using these estimates, the ML estimate of h(t) is $\hat{h}(t) = (0.50)(0.567)t^{-0.433}$. Evaluating $\hat{h}(t)$ at t = 400 we find that the ML estimate of the intensity of distinct Type B modes at the end of test is $\hat{h}(400) = 0.021$.

Crow (1974) shows that conditioned on M = m, the estimate

$$\bar{\beta} = \frac{m-1}{M} \hat{\beta} \quad (44)$$

is an unbiased estimate of β. If we consider

$$\bar{h}(T) = \begin{cases} \frac{m\bar{\beta}}{T} & m \geq 2 \\ 0 & \text{Otherwise,} \end{cases} \tag{45}$$

as an estimate of h(T) then, for Prob $\{M = 0 \text{ or } M = 1\}$ near 0, h(T) is approximately unbiased. In practice, for a complex system under development, the probability of two or more problem failure modes being observed is usually near unity. In this case, it is reasonable to use $\bar{h}(T)$ instead of $\hat{h}(T)$ for estimating the rate h(T).

We will now discuss a procedure for determining a projection of r(T) which is essentially unbiased under the assumptions of the model for estimating B(T) and for Prob $\{M = 0 \text{ or } M = 1\} = 0$. Let

$$\bar{B}(T) = \hat{\mu}_d \bar{h}(T) \tag{46}$$

and consider the projection $\bar{r}(T)$ for r(T) where

$$\bar{r}(T) = 1/T\ (N_A + \sum_{i=1}^{M} (1 - d_i)\ N_i) + \bar{B}(T). \tag{47}$$

For this model, we have approximately that

$$E(\bar{r}(T) - r(T)) = 0. \tag{48}$$

The projected mean time between failure (MTBF) is

$$MTBF = (\bar{r}(T))^{-1}. \tag{49}$$

Example

We illustrate the calculation of the projection $\bar{r}(T)$, utilizing data generated by computer simulation with $\lambda_A = 0.02$, $\lambda_B = 0.1$, $K = 100$ and the d_i's distributed according to a Beta distribution with mean 0.7. For this simulation the system was tested for T = 400 hours and experienced N = 42 failures. Of these failures, there were $N_A = 10$ failures which were Type A and $N_B = 32$ failures which were Type B. In addition, the 32 Type B failures were due to M = 16 distinct Type B modes.

The cumulative test times corresponding to the occurrence of the Type A modes are: 43.16, 49.08, 75.62, 167.27, 238.73, 255.29, 277.33, 350.28, 353.03, 367.68. For the 16 distinct Type B modes, we list mode number and the cumulative failure times for that mode. These are:

Mode 1, 56.42, 72.09, 339.97
Mode 2, 192.66
Mode 3, 47.46, 350.2

Mode 4, 285.01
Mode 5, 379.43
Mode 6, 249.15, 324.47
Mode 7, 133.43, 177.38, 324.95, 364.63
Mode 8, 125.48, 164.66, 303.98
Mode 9, 15.04, 254.99
Mode 10, 111.99, 263.47, 373.03
Mode 11, 53.96, 315.42
Mode 12, 99.57, 274.71
Mode 13, 25.26, 120.89, 366.27
Mode 14, 388.97
Mode 15, 395.25
Mode 16, 100.31

For the Type B modes listed above, the assigned EFs are, respectively, 0.87, 0.70, 0.77, 0.64, 0.72, 0.63, 0.74, 0.89, 0.67, 0.85, 0.77, 0.92, 0.72, 0.69, 0.46, 0.50. The times $X_1 < X_2 < \ldots < X_{16}$ of first occurrence for the 16 distinct Type B modes are: 15.04, 25.26, 47.46, 53.96, 56.42, 99.57, 100.31, 111.99, 125.48, 133.43, 192.60, 249.15, 285.01, 379.43, 388.97, 395.25.

Using Equations (38) and (37), we calculate $\hat{\beta} = 0.797$ and $\hat{\mu}_d = 0.721$. The unbiased estimate $\overline{\beta}$ of β, given by (44), is $\overline{\beta} = 0.747$ and, from (45), the estimated rate of occurrence of new Type B modes at time T is $\overline{h}(T) = 0.030$.

To calculate the projection $\overline{r}(T)$, we also determine, $\overline{B}(T) = 0.022$, using (46), $N_A/T = 0.025$

and $$\sum_{i=1}^{M} (1-d_i) \frac{N_i}{T} = 0.020,$$

where N_i is the number of observed failures for the i-th mode observed, $i=1,\ldots,M$.

We now use Equation (47) to calculate the projected estimate $\overline{r}(T)$. This gives

$$\overline{r}(T) = 0.025 + 0.020 + 0.022 = 0.067$$

and the projected system MTBF is

$$\text{MTBF} = (0.067)^{-1} = 14.9.$$

For the same data, the adjustment procedure estimate $r^*(T)$, from (8), is

$$r^*(T) = 0.025 + 0.020 = 0.045.$$

The corresponding MTBF for the adjustment procedure is $[r^*(T)]^{-1} = 22.2$. This is an estimate of the upper bound on the achievable MTBF for the system and is not an estimate of the current system MTBF. The actual upper bound,

using $(E[r^*(T)])^{-1}$ given by Equation (18) and the parameters of the simulation model, is 20.6. In addition, for this example, it can be stated that 14.9 is an estimate of the current system MTBF. The actual MTBF at time T, given by $[r(T)]^{-1}$ from Equation (2), is 14.7.

4. CONCLUSIONS

In this report, we have shown that the adjustment procedure for reliability projection is biased and, in fact, estimates an upper bound on system reliability after all problem modes have been found and fixed. A model was developed which yields, under reasonable assumptions, an unbiased estimate of the true system reliability after delayed fixes have been incorporated. It is emphasized that the adjustment procedure should not be used for reliability projections.

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